Abstract

This dissertation explores the geometric and algebraic structures of Banach spaces and Banach *-algebras, with particular emphasis on linear and non-linear isometries, as well as algebraic operations such as multiplication, employing techniques from both linear and non-linear Functional Analysis, Banach space geometry, and abstract harmonic analysis.

The work is based on three research papers published in international peer-reviewed journals: two single-authored publications and one co-authored with the supervisor, appearing in *Proceedings of the American Mathematical Society*, *Studia Mathematica*, and *Proceedings of the Royal Society of Edinburgh Section A: Mathematics*.

A central theme of the dissertation is the analysis of self-isometries in Tsirelson-type spaces, particularly the combinatorial Tsirelson spaces $T[\theta, \mathcal{S}_\alpha]$, where $0 < \theta < \frac{1}{2}$ and $\mathcal{S}_\alpha$ denotes the Schreier family of order α, for $1 \leq \alpha < \omega_1$. A complete description of linear surjective isometries in this class is provided, and the results are further applied to address Tingley's problem in this specific setting—a longstanding question concerning the extension of sphere isometries to the entire space.

The results contribute to the growing body of literature seeking to resolve Tingley's problem also beyond classical Banach spaces.

In a related direction, the dissertation also examines the openness of multiplication in commutative, smooth Banach *-algebras and its relation to the covering dimension of the underlying compact space. In collaboration with my supervisor, general conditions under which multiplication is open have been established, and a complete characterisation of (uniform) openness in algebras of continuous complex–valued functions is provided in terms of covering dimension. It is also demonstrated using ultrapower techniques that certain structural properties of discrete groups—such as having elements of arbitrarily large order—prevent uniform openness of convolution in their ℓ_1-group algebras.

Taken together, these findings highlight a common theme: the stability of functional structures in Banach spaces under linear, non-linear, and bilinear transformations.

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