Abstract

Title: Weighted shifts on directed trees and weighted composition operators
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In this thesis we deal with three different problems concerning weighted shift operators and composition operators. The first one is the completion problem for weighted shifts on directed trees: the question whether it is possible to complete the weights in a way to obtain an operator satisfying further conditions. We mainly focus on subnormal and completely hyperexpansive completion problem. Since these classes of operators are characterized by certain moment sequences, in our research we deal with truncated moment problem on the intervals $(0, \infty)$ and $(0, 1]$. In Chapter 2 we present general results in this subject; in particular, we describe k-step backward extensions of truncated moment sequences. As an application, in Chapter 3 we give the full solution to the subnormal and completely hyperexpansive completion problem for weighted shifts on directed trees with one branching point.

The second problem is the characterization of m-isometric composition operators on discrete spaces. In Chapter 4 we present how such operator can be viewed as a weighted shift on certain graph and we give a full classification of these graphs. Next, we show the description of m-isometric composition operators for one class of graphs, namely, for graphs with one cycle. These results are then applied to solve the Cauchy dual subnormality problem for 2-isometric operators.

In Chapter 5 we deal with bilateral weighted shifts with operator weights. Our purpose was to establish a general characterization of unitary equivalence of such shifts in the case when weights are quasi-invertible. Among several application of this result we emphasize the following theorem: if the weights defined on $\mathbb{C}^2$ are positive, then under further assumptions the unitary equivalence can be always given by the operator having at most two non-zero diagonals.

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