Report on Kenne thesis

Pseudo Leja sequences associated to a compact set $K \subset \mathbb{C}^n$ are natural generalizations of Leja sequences which were introduced in [11]. The same proof that Leja sequences are "asymptotically Fekete" (cf., [17]) in the sense of recovering the transfinite diameter $D(K)$ of K works to show the same is true for pseudo Leja sequences of Edrei growth (Definition 2.1.1). This is Theorem 2.2.1 of the thesis, which is the main result of the author's paper *Multidimensional pseudo Leja sequences* appearing in Volume 17 (2024) of the *Dolomites Research Notes in Approximation*, pp. 72-81. The essential theoretical content of the thesis is contained in Chapter 3 on intertwining pseudo Leja sequences. Intertwining two Leja sequences on $\mathbb{C}$ to get a natural sequence of points on $\mathbb{C} \times \mathbb{C}$ was done by Schiffer and Siciak (1962) in order to prove a product formula for the transfinite diameter $D(K)$ of a product set $K = K_1 \times K_2$, $K_1, K_2 \subset \mathbb{C}$. Calvi and Phung (*Bull. Pol. Acad. Sci. Math.* (2005)) generalized this to $K = K_1 \times K_2$, $K_1 \subset \mathbb{C}^{n_1}$, $K_2 \subset \mathbb{C}^{n_2}$ using intertwined "block" Leja sequences. I'm surprised these papers are not referenced, but I suppose more relevant to this thesis is [37] where Calvi constructs unisolvent arrays on $\mathbb{C}^p \times \mathbb{C}^q$ by interwtwining appropriately ordered unisolvent arrays on each of $\mathbb{C}^p$ and $\mathbb{C}^q$. In the thesis, Kenne uses linear algebra to show Theorem 3.2.1 which generalizes a result of Irigoyen [60]. Precisely, if $A \subset K_1 \subset \mathbb{C}^{n_1}$ and $B \subset K_2 \subset \mathbb{C}^{n_2}$ are appropriate sequences in compact sets K_1, K_2, and $\Omega := A \otimes B$ is their intertwining in $\mathbb{C}^{n_1} \times \mathbb{C}^{n_2}$, then Ω is a pseudo Leja sequence for $K_1 \times K_2$ if and only if A and A and B are pseudo Leja sequences for K_1 and K_2 (modulo the hypothesis that K_1, K_2 are polynomial determining compacta).

Chapters 4, 5 and 6 are mostly numerical in nature. Chapter 4 gives an algorithm for computing discrete Leja points and numerical examples. Chapter 5 is essentially reference [13] while Chapter 6 is essentially reference [14], both of which have already appeared in print. In fact, Chapter 7 is also the content of another of the author's papers, *A new estimate of the transfinite diameter of Bernstein sets.* This appeared in *Math. Inequal. Appl.* 27 (2024), no. 3, 735-743. A Bernstein set is a set satisfying a Markov inequality with exponent one. Since the result, Theorem 7.1.1, is theoretical in nature, I will make one comment: the improvement in his upper bound of the transfinite diameter of a

Bernstein set from previous results arises from the author's interesting observation that one can get a very precise Markov inequality for "monic-like" polynomials $e_j(z) + \sum_{i<j} c_i e_i(z)$ where $e_0, e_1, e_2, \ldots$ is an ordering of the monomials.

Given that essentially the whole thesis, modulo the introduction, has appeared in reasonable journals, the body of work certainly satisfies the requirements of a PhD thesis at Jagiellonian University.

Sincerely,
Norman Levenberg

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