

# Abstract

A topological dynamical system is a pair  $(X, T)$  consisting of a compact metric space  $X$  and a continuous self-map  $T$  of  $X$ . The main goal of the thesis is to examine the dynamical properties of  $(X, T)$  related to the well-known Besicovitch pseudo-metric  $D_B$ . We study limits of sequences of generic points with respect to the Besicovitch pseudo-metric  $D_B$ . Using the characterization of the spectrum of an ergodic measure via its generic points in [3], we analyze the properties of the (limit) measure by passing to the limit in  $D_B$ . Specifically, we show that the set of generic points for discrete spectrum, totally ergodic, (weakly) mixing, and zero entropy measures forms a closed set with respect to the Besicovitch pseudo-metric. In addition, a pseudo-metric  $\bar{d}$  on  $(A^\infty, \sigma)$  is uniformly equivalent to the Besicovitch pseudo-metric. Using the techniques in [26] and the pseudo-metric  $\bar{d}$  we study shift spaces that are  $\bar{d}$ -approachable and have the  $\bar{d}$ -shadowing property. We analyze the relationship between the notions of  $\bar{d}$ -approachability,  $\bar{d}$ -shadowing, and  $\bar{d}_{\mathcal{M}}$ -stability. We prove that the  $\bar{d}$ -shadowing property implies  $\bar{d}_{\mathcal{M}}$ -stability. We also investigate connections between Hausdorff pseudo-distances  $\bar{d}^H$  (for shift spaces) and  $\bar{d}_{\mathcal{M}}^H$  (for simplices of invariant measures). We prove that if two simplices of invariant measures are close in  $\bar{d}_{\mathcal{M}}^H$  and both shift spaces have the  $\bar{d}$ -shadowing property then shift spaces must be close in  $\bar{d}^H$ . We also provide proximal and minimal examples of  $\bar{d}$ -approachable shift spaces. Both examples have the  $\bar{d}$ -shadowing property and positive topological entropy, and they are mixing. We also study connections between the asymptotic average shadowing property (a variant of the classical shadowing property) and the vague specification property (a variant of the specification property) for general topological dynamical systems. Using the Besicovitch pseudo-metric and its relatives we show that the asymptotic average shadowing property and the vague specification property are equivalent. Finally, we show that the proximal example of  $\bar{d}$ -approachable shift space has the vague specification property using the notion of Besicovitch-completeness.

Melih Emin Can  
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