PhD dissertation summary

Classification problems in topological dynamics and ergodic theory

The overarching topic of this thesis is the study of certain questions at the intersection of topological dynamics and descriptive set theory. The work presented can be divided into three main parts.

In Chapter 3, we consider a topological dynamical system (X, T) and an invariant Borel probability measure μ on X. Let $Gen(\mu)$ be the set of points generic for μ. It turns out that $Gen(\mu)$ is always a Borel set, in fact a Π^0_3 set. What is the possible Borel complexity of $Gen(\mu)$? We consider this question under the assumption that the system is transitive or minimal. Here by *Borel complexity* of a set, we mean the lowest stage of Borel hierarchy that it belongs to. We find some constraints on the possible Borel complexity of $Gen(\mu)$. We give a multitude of examples of minimal systems and transitive systems together with measures that yield certain Borel complexities. Next we consider systems (X, T) that have a certain variant of the specification property. Under this assumption, if (X, T) is not uniquely ergodic, then $Gen(\mu)$ is Π^0_3-complete for every T-invariant Borel probability measure μ on X. We obtain analogous results for other sets of points described by statistical behaviors along their orbits (e.g. the quasigeneric set and the set of maximal oscillation). Some results of this chapter appeared in a joint paper of the author with Kwietniak, Jackson and Mance.

In Chapter 4, we consider the isomorphism relation between certain classes of subshifts in the framework of Borel reducibility of countable Borel equivalence relations. We prove that the isomorphism relation for subshifts with the specification property is not amenable. This can be interpreted as an answer to Bowen's Problem 32. We prove that the isomorphism of pointed transitive subshifts is not amenable.

In Chapter 5, we study the complexity of conjugacy and flip-conjugacy of Cantor minimal systems. We determine that both equivalence relations are complete analytic, and therefore not Borel. This answers a question of Gao. This is a result of joint work with García-Ramos, Kasprzak, Kunde, and Kwietniak, not yet published.

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