Summary of the doctoral thesis

In this dissertation we use the computer assisted methods to investigate dissipative partial differential equations. We develop our methodology for two problems:

- the problem governed by the Brusselator system of two nonlinearly coupled PDEs,
- the problem governed by the non–autonomous Chafee-Infante problem.

For the Brusselator system with diffusion and Dirichlet boundary conditions on one dimensional space interval we prove with the aid of computer that, for certain values of the problem parameters, the system has a periodic orbit. Additionally, we present the numerical evidence of the occurrence of the period-doubling bifurcation and invariant torus in this system.

For the Chafee-Infante problem we prove that, for some rage of parameters, the periodic orbit exists. In particular we present the existence of such solutions in the case when non linearity can change sign, and theoretical methods, even for global existence of solutions may fail. Moreover we prove that the constructed orbit is locally attracting with respect to the supremum norm.

We present C^0 and C^1 algorithm for rigorous integration of dissipative partial differential equations, which we use in the in computer assisted proofs. The C^0 algorithm is used to obtain the existence of periodic orbits, and C^1 algorithm for their attractivity. In particular we develop a new method of the variable aggregation consisting on the estimates of the evolution of the flow on the unbounded sets. This method is useful to solve rigorously the variational equation simultaneously for the infinite number of initial data.

The structure of the dissertation is as follows. Section 1 introduces the topic of computer assisted proofs for partial differential equations. We discuss the main results, used methods, and the possible extensions and application of this work. In Section 2 we briefly recall needed facts from dynamical systems, theory of integrals in Banach spaces, and linear equations in Banach spaces. In next Section 3 we describe the theory of nonlinear abstract equations in the Banach spaces and the theory of variational equations for them. Section 4 is devoted to the description of C^0 and C^1 integration algorithms for the abstract problem formulated in the previous section. In Section 5 we describe the computer assisted proof of existence of periodic orbits for Brusselator system with the use of C^0 algorithm of integration. Section 6 contains numerical and rigorous results for various parameters of the Brusselator system. In Section 7, in turn, we describe the computer assisted proofs for the Chafee–Infante system. Finally, the Section 8 is the Appendix which contains some auxiliary inequalities and estimations. The main part of it is Section 8.2 which contains the results on the algebra of infinite series utilized in the algorithms.

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