Summary of the doctoral thesis

The main goal of the thesis is to present applications of graded Kac-Moody Lie algebras in three distinct settings: as a source of interesting classes of Koszul modules and resonance varieties; as a tool for constructing equivariant embeddings of homogeneous spaces; and as they appear in the structure theory for Gorenstein ideals of codimension four.

We introduce Koszul modules associated with graded Kac-Moody Lie algebras and study their properties. The main result is the characterization of nilpotent Kac-Moody Koszul modules. We give an elementary condition, stated in terms of generalized Dynkin diagrams, equivalent to their nilpotency. We also give a precise description of all Kac-Moody Koszul modules of finite length. We determine equivariant resonance varieties for exceptional types G_2, F_4, E_6 and E_7 and homological invariants of the corresponding Koszul modules. We also present some results on homological invariants of generic Koszul modules.

In the second part of the thesis, we focus on embeddings of homogeneous spaces for Kac-Moody groups into Grassmanians of irreducible representations. Let $\mathcal{P}$ be a maximal parabolic subgroup of a Kac-Moody group $\mathcal{G}$ and let $V(\lambda)$ be the irreducible representation with the highest weight λ. We prove the existence of an embedding $\mathcal{G}/\mathcal{P} \hookrightarrow \mathrm{Grass}\,(d, V(\lambda))$ for a certain natural number d, and study its properties. In particular, we focus on fundamental representations and simply-laced Dynkin diagrams A_n, D_n and E_6.

The last part of the thesis deals with minimal free resolutions of Gorenstein ideals of codimension four. Differentials and spinor coordinates form the first level of the associated *higher structure maps*, generalizing the theory of Buchsbaum-Eisenbud multipliers. The higher structure maps appear in graded components of critical representations, which generate a generic ring for the truncated resolution. We study higher structure maps of ideals with six generators in more detail. We compute the general formulas, as well as some important special cases. The main application is a proof of a long-standing conjecture: every Gorenstein ideal of codimension four with six generators is a hypersurface section of a codimension three Gorenstein ideal.

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