Michał Farnik Kraków, 01.03.2023

Abstract of the Ph.D. thesis
"A hat guessing game"

In our thesis we study a variant of a well known hat guessing game. In the game there are n players and an adversary. The adversary places a hat of one of K colors on the head of each player. The players are placed in vertices of a loopless graph G, two players can see each other's hats if and only if they are placed in vertices adjacent in G. No other interaction is allowed. Each player guesses privately up to s times what hat he is wearing. The goal of the players is to ensure that at least one of them guesses correctly. To this end they are allowed to meet before the hats are placed and determine a public deterministic strategy. The players know the graph G and their placement before determining their strategy. We say that the players have a winning strategy for graph G, s guesses and K colors if they have a strategy such that for every hat placement there is at least one player who guesses correctly. We define the s-hat guessing number HG(s,G) of a graph G which is the largest integer K such that the players have a winning strategy for G, s and K.

Using Lovász local lemma and entropy compression we show that HG(s,G) = O(sΔ(G)). We also examine the relation of HG(s,G) and col(G). We show that for a tree T we have HG(s,T)≤s^2+s. Also for each n, m = m(n) big enough and a complete bipartite graph $K_{n,m}$ we have HG(s, $K_{n,m}$)=ns^2+s.We conjecture that HG(s,G) is bounded by a function of s and col(G) and prove this for a special kind of guessing strategies. We prove a lemma which allows the adversary to divide a graph G into subgraps G1 and G2 and to find a winning strategy in a game on G using winning strategies in games on G1 and G2. We use this lemma to bound the s-hat guessing number for graphs with large girth.

Michał Farnik