Institute of Mathematics of the AS CR, v.v.i.
Žitna 25, 115 67 Praha 1
Czech Republic

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Report on the thesis "Stability theory for matrix polynomials in one and several variables with extensions of classical theorems" by Oskar Jakub Szymański

The thesis is devoted to the study of matrix-valued polynomials with eigenvalues localized in a certain given subset of the complex plane. The key new notion is that of hyperstability, which is stronger than the stability (i.e., the localization property of eigenvalues). On the other hand, it is weaker than another important property – the localization of the numerical range of the matrices, which are the values of the polynomial function. In some situations the notion of hyperstability is a proper assumption and proves to be useful.

The main inspiration for the research were two classical results – the Gauss-Lucas theorem giving the relation between the roots of a polynomial and the roots of its derivative, and the Szász inequality, which gives an estimate of a stable polynomial in terms of its few first coefficients.

The dissertation thesis is divided into seven chapters. The first chapter is an introduction and gives a survey of known facts. The second chapter introduces the notion of hyperstability both in single variable and multivariable setting. The chapter gives summary of the basic properties and shows its relations with the other related notions mentioned above.

The third chapter is devoted to the generalization of the Gauss-Lucas theorem to matrix polynomials. The main results are Theorems 3.3 and 3.12., which use the assumption that the entries of the matrix polynomial are linearly independent polynomials.

The next chapter gives various generalizations of the Szász inequality in the matrix case. In general, the (hyper)stability is not sufficient for proving Szász-type estimates for matrix-valued polynomials. There are some results of this type using rather strong assumption localizing the numerical range of the matrices. Here another approach is used. Instead of conditions on the numerical range it is assumed that the matrix polynomial can be factorized in a proper way.

Tel.: +420-222 090 711
Fax.: +420-222 090 701
E-mail: mathinst@math.cas.cz
www: http://math.cas.cz
Bank Connection.: CNB Praha
Account No: 69623011/0710
Company Identif. No: 67985840
Tax Identif. No: CZ67985840

The fifth chapter gives various methods of producing a hyperstable polynomial from two-variable stable quadratic or cubic polynomials.

The next chapter describes various operations on polynomials preserving the hyperstability. In the last chapter various applications of the results from the previous chapters are given.

A natural question for further research seems to be to what extend the obtained results can be generalized to the infinite-dimensional setting, for operator-valued polynomials. Clearly the eigenvalues would have to be replaced by the spectrum (or approximate point spectrum).

Most of the results presented in the thesis were published in [23], the joint paper of the author with his supervisor. The paper was published in 2023 in Lin. Alg. Appl., which is a mathematical journal of a good quality. Some other results appear in the preprint [18], published on Arxive by the author, the supervisor and P. Pikul.

The dissertation thesis would be more convincing if there existed more publications of the author, preferably without the supervisor. Nevertheless, the thesis presents new interesting results published in a respected mathematical journal. I recommend that the thesis be accepted for a successful defense.

Vladimír Müller
Institute of Mathematics, Czech Academy of Sciences
Žitná 25, 115 67 Praha 1
Czech Republic

Tel.: +420-222 090 711
Fax.: +420-222 211 638
E-mail: mathinst@math.cas.cz
www: http://math.cas.cz
Bank Connection.: CNB Praha
Account No: 69623011/0710
Company Identif. No: 67985840
Tax Identif. No: CZ67985840