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To Whom It May Concern:

This is a review of the Dissertation entitled "Stability theory for matrix polynomials in one and several variables with extensions of classical theorems" by Oskar Jakub Szymański, a Ph.D. candidate at Jagiellonian University.

My overall assessment of the dissertation is that it indeed satisfies the requirement that it constitutes an "original solution to a scientific problem." The dissertation explores the new concept of *hyperstability* at length. The concept is natural and while many of the proofs and theorems flow easily from scalar proofs, the dissertation presents many examples to demonstrate subtleties of the condition and places where scalar results do not apply. The section on the Szász inequality for matrix functions is also quite interesting and novel. I appreciated the paper's thorough proofs which I believe is necessary in a Ph.D. dissertation. A large portion of the dissertation is already published in a well-regarded and peer reviewed journal, and therefore I have no hesitation in recommending the approval of this dissertation.

I am not certain if it is appropriate (since I am uncertain if revisions are allowed at this point), but I have included a list of suggestions both substantive and minor on an attached page.

Sincerely,



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SUBSTANTIVE SUGGESTIONS

- (1) Page 8. Reference [11] is to an unpublished preprint from 2012. I would be a little uncomfortable simply citing such a paper and I think some remark should be made about this. Should the validity of the paper be trusted? Should the dissertation include the proof? I believe the dissertation proves something stronger later so there is nothing to worry about but one should be cautious here.
- (2) Page 9. Here there is a reference [28] to online lecture notes. These could be incorrect or disappear at any time so I would be reluctant to cite this. Perhaps a published reference could be found. (I do not know off the top of my head but perhaps a paper/survey on hyperbolicity could have this fact available.)
- (3) Page 36. Proposition 4.5. The factorisation of P is quite restrictive and I think something should be said to justify studying such a special case. I realize this case applies to the functional calculus results later but still.
- (4) Page 40. The inequality $\log(n+x) \leq x+n-1$ is not a great substitute for $\log(1+x) \leq x$. Perhaps the concavity inequality $\log(n+x) \leq \frac{1}{n}x + \log(n)$ would get a much better result?
- (5) Page 43, remark 4.13. The remark says that the proof of the proposition proves $|a_1|^2 - 2\Re a_2$ is non-negative but isn't this rather a result of Lemma 4.4?
- (6) Page 57. Example 6.3. The example shows local uniform limit of hyperstable polynomials need not be hyperstable. Is there any hope to characterize the local uniform limits of hyperstable polynomials?
- (7) Chapter 7. No motivation is given for the detailed low degree examples presented in this chapter. I believe some motivation is given the published paper of the author. Giving motivation for these examples would greatly improve the dissertation.

MINOR SUGGESTIONS

- (1) Numerous spelling errors were found in the dissertation; this could easily be solved by running spell check.
- (2) Page 7, 1st paragraph: "as to scalar polynomials" should be "as scalar polynomials". Likewise, "as to matrix polynomials" should be "as matrix polynomials".
- (3) Page 7, paragraph 3. "we come with interesting example" should be "we present an interesting example"
- (4) Example 1.9. There should be some non-degeneracy condition to prevent p from being identically zero.
- (5) Page 20, proof of Proposition 2.5. There is a reference to equation (2.5) which should reference (2.2).
- (6) Page 23. "Laplace's determinant formula"
- (7) Page 23, proof of Theorem 2.10. Why does the proof switch from using λ to using μ ?
- (8) Page 24. "module" should be "modulus"
- (9) Page 27. 'linear independent polynomials' -> 'linearly independent polynomials'
- (10) Page 28, 4th line from bottom. 'than' should be 'then'.

- (11) Page 29, remark 3.9. I'm a little unsure why the Sendov conjecture is brought up, since not much is said about it.
- (12) Page 38. "Theorem 4.5" should be "Proposition 4.5"
- (13) Page 43. "where the but last one" would sound better as "where the second to last"
- (14) Page 61. Beginning of proof of prop 7.1. "It was showed" should be "It was shown"
- (15) Bibliography. There are various capitalization mistakes that can be fixed by putting braces around capital letters.