Referee's report on the PhD thesis "Application of adaptive grids in basket option pricing" by Mr Rafał Muchorski

This report is prepared at the request of the Chairman of the Mathematics Discipline Council, Faculty of Mathematics and Computer Science, Jagiellonian University, letters dated 8 March 2022 and 4 May 2022.

The thesis tackles the challenging problem, both theoretically and computationally, of pricing European derivative securities written on a weighted basket of correlated underlying assets (that is, so-called basket options).

In the theoretical part of the work the candidate presents and analyses a model of the underlying assets based on a multivariate shifted log-normal mixture distribution of forward prices. This is similar to the multivariate mixture dynamics model of Brigio *et al.*[1], but with modifications of the original version which improve tractability without sacrificing the flexibility of the model to match input data. The choice of the mathematical model of the forward prices of the underlying assets appears to be a well-balanced compromise, as demonstrated by the theoretical and numerical results established in the course of this PhD and expounded in the thesis. European basket option prices are then expressed in terms of expectations under the corresponding forward measure, and these expectations can in turn be evaluated as multidimensional integrals.

What follows is a novel adaptive grid technique developed by the candidate for the numerical approximation of such multidimensional integrals, hence of basket option prices, culminating in Theorem 1, the principal theoretical result of the thesis, demonstrating the convergence of the adaptive scheme.

This part of the dissertation, devoted to the theoretical foundations of both the mathematical model and the numerical work implemented in the latter part of the thesis, provides ample evidence of skilful mathematical workmanship, both in respect of mathematical modelling as well as formulating and proving highly non-trivial relevant mathematical results.

Having established a sound theoretical basis, the author proceeds to numerical work based on approximating specific multidimensional integrals by linear combinations of one-dimensional integrals obtained by means of a suitably chosen adaptive scheme with the smallest possible adaptive error under a constraint on the number of grid points. This delivers an approximation of basket option prices, which can be expressed in terms of such multidimensional integrals.

Calibration of the model of the underlying assets to market data is also discussed.

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[1] D. Brigo, C. Pisani, and F. Rapisarda, The multivariate mixture dynamics model: Shifted dynamics and correlation skew, Annals of Operations Research, 299, 1411-1435 (2021).

In the concluding chapter of the thesis a number of numerical experiments are reported for a basket option with six underlying assets under various parameter choices, comparing the adaptive scheme technique with two other benchmark approaches, namely the Monte Carlo method with variance reduction by means of control variables and Quasi-Monte Carlo based on multi-dimensional low-discrepancy Sobol sequences.

Conclusion: Mr Rafał Muchorski's thesis demonstrates the candidate's maturity as a mathematician at a level comparing favourably with some of the best PhD dissertations I have recently refereed and examined at various institutions (e.g. the University of Sydney, Manchester University, Imperial College London, the University of Leeds, or the University of York). Mr Muchorski's work constitutes a significant scientific contribution towards the long-standing problem of efficient pricing of basket options. The thesis meets and in various respects exceeds the requirements for a PhD in mathematics. I therefore recommend that the candidate should be awarded a PhD degree with distinction.

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