Application of adaptive grids in basket option pricing

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Summary of the PhD thesis

The PhD thesis concerns the application of a new numerical method to price basket options. In first place a multidimensional local volatility model is introduced to model forward prices of assets. In this model the random vector of asset forward prices has a mixture distribution, where each distribution from this mixture has log-normal marginals (with a possible shift) and a Gaussian copula explaining asset price dependence.

We derive formulas for the theoretical, arbitrage-free price of a European basket option, which can be expressed as a certain multidimensional integral. It does not possess an explicit analytical formula. To numerically approximate its value we propose a novel quadrature method based on the concept of "adaptive grids".

Integration in n dimensions, which is related to the basket option price formula, can be expressed as an $n-1$-dimensional integral, where the integrand is a one-dimensional integral and is a function of $n-1$ variables. For such function we construct a one dimensional quadrature with approximation error (adaptive error) depending on first and third partial derivative. We next generalize the one-dimensional quadrature to the multidimensional case. The $n-1$-dimensional quadrature evaluates the integrand in points from the $n-1$-dimensional adaptive grid.

An appropriate application of the grid enables to achieve adaptive properties of presented quadrature rules. First property is the control of adaptive error by selecting appropriate parameters related to adaptive grid used in approximation. Secondly, the adaptive error is numerically tractable and uniform in respect to the parameter corresponding to the strike of the basket option. The resulting price approximation is expressed as a weighted sum of prices from Black's formula. Considering the approximation error such approach provides an efficient method for calibrating implied volatility and implied correlation defined for a basket option. Aforementioned properties can be utilized within a model calibration that uses quotes of (single asset) plain-vanilla options and options on baskets of those assets (ex. index options), and price such instruments consistently.

Finally, we present numerical results by testing our presented method with standard methods used in basket option pricing, such as Monte Carlo with variance reduction and quasi Monte Carlo using low discrepancy sequences. Tests performed for representative sets of parameters and reasonable numerical restrictions prove usefulness of adaptive grids in practical applications.

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