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PhD Committee
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Report on the PhD thesis of Patryk Mikos
Geometric and weight constraints in Online Interval Coloring

Overall evaluation. The presented thesis is a solid dissertation, clearly exceeding minimum requirements for a PhD degree in theoretical computer science.

The thesis contains numerous result around the structure of interval graphs, with particular emphasis of the online coloring problem. The author mastered previous techniques and approaches from the literature, cleverly modified and combined them to obtain a number of results interesting for the international audience. In particular, I would like to praise here his indepth understanding of the lower bound techniques.

Notably, most of the results appeared as single-authored or co-authored with the co-advisor publications or preprints. It proves independence and high quality of skills of Mr. Mikos.

Finally, the quality of the write-up is very high and the thesis is well-organized.

Consequently, the presented thesis satisfies all the requirements to award a PhD degree to the candidate.

Overview of the thesis. The main topic of the thesis are interval graphs, in particular online colorings in (subclasses of) this graph class.

An *intersection model* of a graph G is an assignment of a set A_v to every vertex $v \in V(G)$ such that $A_v \cap A_u \neq \emptyset$ if and only if $uv \in E(G)$ for every distinct $u, v \in V(G)$. A graph G is an *interval graph* if and only if G admits an intersection model where every set A_v is a closed segment on a line. Interval graphs are one of the most studied and important subclasses of chordal graphs, modelling various scenarios from applications.

In the *online coloring problem*, one learns vertices of a graph G one-by-one and, as a vertex arrives, it needs to assign it a color $f(v)$ that is distinct than the colors of the previous neighbors of v. The quality of the algorithm is measured by how many colors it uses compared to the (offline) chromatic number of the final graph. The *competetive ratio* is the worst-case ratio of these numbers and *asymptotic competetive ratio* is the limes superior of these ratios as the chromatic number goes to infinity.

For interval graphs, a celebrated work of Kierstead and Trotter shows an algorithm using at most $3k - 2$ colors on an interval graph of chromatic number k. Furthermore, they showed that this is tight: there is an adversarial strategy for presenting a k-colorable interval graph to the algorithm that always forces it to use at least $3k - 2$ colors.

This motivates the following questions, pursued in the thesis:

1. What happens in the *bandwidth* case, where every vertex (interval) has a bandwidth $b_v \in (0, 1]$ and the coloring constraint is that any monochromatic clique needs to have total bandwidth at most 1?
2. What happens for unit interval graphs, where all intervals in the model have the same length? Here, it makes a difference if the algorithm is presented with the graph only (i.e., vertex and its adjacencies) or with the interval model. Furthermore, it makes also a difference if the presented model is "unit interval" (all intervals of the same length) or "proper interval" (no interval properly contained in the other) — both definitions define the same graph classes, but the "proper interval" variant has more flexibility (and hence in some scenarios gives more power to the adversarial presenter).
3. What happens between unit interval graphs and interval graphs, when

we bound the ratio of the longest-to-shortest interval in the model?

4. How does FirstFit, a natural heuristic, behaves in the scenarios above?

Finally, the candidate addresses also a related question of enumerating all interval graphs of fixed size with small delay.

Bandwidths. Here, the authors studies variant where the algorithm is presented full interval model of the graph.

For general bandwidths, the author improves two lower bounds from the literature: for general interval graphs, the lower bound for asymptotic competetive ratio of $\frac{24}{7}$ is improved to 4.1626 and for unit interval graphs from 1.831 to 2.

Both results are obtained through essentially the same construction, stemming from a deep understanding of the tricks used in the lower bound of Kierstead and Trotter. The author very skillfully develops a way to elegantly iterate the main Kierstead-Trotter trick in the bandwidth regime, with computer-assisted finding of best parameters.

Different representations of unit interval graphs. Here, similarly as in the previous part, the author presents a set of lower bounds for problems with bandwidths, significantly improving the bounds known from the literature. The lower bounds are: $2\frac{653}{994}$ for graph representation, $2\frac{1}{4}$ for proper interval models, and 2.1571 for unit interval models.

The technique is again a skillful interplay of a few tricks from previous works, including Kierstead-Trotter separation construction and a lower bound of Epstein and Levy for the non-bandwidth case. And again, finding the best parameters for the constuction requires computer-assisted analysis.

Bounded aspect ratio of intervals. The question here is: to what extend one can improve the Kierstead-Trotter bounds if one assume that the presented interval model has only intervals of lengths from $[1, \sigma]$ for a parameter σ? The lower bound of Kierstead and Trotter requires arbitrarily large intervals (larger as the chromatic number goes to infinity).

First, inspired by recent algorithms for unit disk intersection graphs, the author presents a simple $(1+\sigma)$-competetive algorithm. This algorithm has best known competetive ratio for small values of σ.

Second, the author presents a family of lower bounds agains various values of σ. To achieve this, the author uses most of the insight and tricks from the previous part, together with a new way of iterating the construction.

FirstFit. Here, the author deals with the analysis of FirstFit, the most common heuristic for the online coloring problem. In my opinion, the main result here is an improved analysis of unit interval graphs (asymptotic competetive ratio improved from 6 to 5).

Apart from this result, the author provides a tour over the scenarious studied in previous section and scenarios with different constaints on the allowed bandwidths (it makes a difference if very small or close-to-1 bandwidths are allowed or not).

In this and the previous parts of the thesis, the main weight of the results is in lower bounds, always being a very skillful play with previous ideas, a few new ideas, and a careful choice of parameters for interactions between different parts of the construction.

I would paise these results for two reasons. First, there are quite exhaustive; the author looked under every stone, not necessarily making a final tight answer everywhere, but at least providing new insights. Second, the author really mastered the lower bound toolbox and makes substantial improvements upon state-of-the-art.

Enumeration. The last part of the thesis, on enumeration of interval graphs, is a bit further from the main topic. The goal here is to enumerate all non-isomorphic interval graphs on n vertices and the quality measure is the worst-case time spent between two consecutive outputs. The author improves the known $O(n^4)$-delay algorithm to $O(n^3 \log n)$ by adding new uses of the MPQ-tree datastructure to the algorithm.

This results shows that (a) the author understood the state-of-the-art and bottlenecks there, and (b) deeply understood the structure of interval graphs represented by MPQ-trees. Albeit quite incremental, it is a nontrivial and interesting result.

Summary. The overall evaluation of the thesis has been presented at the begining of the report. Repeating myself, the thesis provides an exhaustive tour on the topic of online colorings of interval graphs, with a number of interesting results, mostly from the lower bound perspective. The thesis shows great skills of the author in the studied area.

On the negative side, one could mention that althought the topics studied in the thesis are of interest to the international community and improve upon a number of papers published at prestigous venues, the studied variants (bandwidths, bounded aspect ratio) are a bit more specialized. The author cannot claim solving any really "hard nut" of central interest to the scientific community and, probably as a consequence, publishes so far his works in second-tier venues.

Despite the above reservation, the thesis, excellently presented, satisfies the requirements for a PhD degree with significant margin.

Yours faithfully,

Marcin Pilipczuk