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To whom it may concern

This is a letter of support for Dr. Sławomir Rams's application for the title of professor.

Sławomir Rams's field of study in mathematics is algebraic geometry. More precisely, he specializes in the study of algebraic subvarieties of the complex projective space that contain either many lines, or many singular points of specified types.

It has been known for two hundred years that surfaces defined by a quadratic (that is, degree-2) equation contain infinitely many lines (this is why cooling towers in nuclear plants are hyperboloids of one sheet). When the surface is smooth and the degree d of its equation is at least 3, the surface contains only finitely many lines, and it has been a challenge for the past hundred years to find the maximal possible number of these lines.

When $d = 3$, this number is 27 and has been known since 1849 (Cayley–Salmon). When $d = 4$, Schur discovered in 1884 a smooth quartic surface with 64 lines and Segre thought he had proved in 1943 that this was the maximal possible number, but his proof was wrong. It is only in 2012 that Sławomir Rams and Matthias Schütt were able in a joint work to fix the gap in the proof and to prove that the maximal number of lines on a smooth quartic surface is indeed 64. Their proof works over any field of characteristic other than 2 and 3. The case where the characteristic of the field is 3 was solved in 2014 (and published the year after in the very good journal *Mathematische Annalen*; the maximal number is 112) and the case where the characteristic of the field is 2 in 2015 (the maximal number is 60), both again by Sławomir Rams and Matthias Schütt, with the final touch in the characteristic-2 case put by Alexander DEGTYAREV with the aid of a computer.

Sławomir RAMS and Matthias SCHÜTT have started to look at the case of quintics ($d = 5$): they just wrote an eprint on arXiv where they show that smooth quintic surfaces contain at most 126 lines.

Smooth quartic surfaces are K3 surfaces and Sławomir RAMS also studied configurations of rational curves on a closely related family of surfaces, that of Enriques surfaces. The various possible configurations give rise to interesting algebraic codes and Sławomir RAMS studied them in a joint article with Matthias SCHÜTT written in 2014 and accepted for publication.

Another interesting part of Sławomir RAMS's work concerns the topological entropy of automorphisms of algebraic surfaces, which, by work of Mikhaïl GROMOV, can be computed from their dynamical degree, the largest (real) eigenvalue of the action of the automorphism on the Néron–Severi group of the surface. These dynamical degrees are either quadratic integers or Salem numbers.

The case of K3 surfaces has been well researched by numerous mathematicians who produced very interesting examples and results. For example, Curtis MCMULLEN was able to compute the smallest nontrivial dynamical degree for K3 surfaces. The case of Enriques surfaces is much more difficult. Sławomir RAMS was able to produce a new constraint on the Salem numbers that are dynamical degrees of automorphisms of Enriques surfaces and he gave a list of 39 possible Salem numbers which contains the smallest nontrivial dynamical degree for Enriques surfaces. I find this a very interesting result (just published in the journal *Mathematische Nachrichten*) because of the various fields that it is related to (arithmetic, geometry, lattices, dynamics).

A problem very much related to finding projective algebraic varieties with many lines, and one which has an equally long history, is finding projective algebraic varieties with many singular points of a simple type (say, nodes or cusps, or more generally A-D-E singularities). Sławomir RAMS proved for example (in collaboration with Remke KLOOSTERMAN) that, under a suitable assumption, a quintic threefold cannot contain more than 10 triple points (and they construct an example with exactly 10 such points). In dimension at least 3, the position of the singular points is very important and is related to the factoriality of various varieties attached to the situation and their Hodge numbers. These kind of questions form the core of Sławomir RAMS's habilitation thesis (published in 2008).

More recently, together with Sławomir Cynk, Sławomir Rams proved formulas for the Hodge numbers of some resolutions of certain 3-dimensional hypersurfaces with A-D-E singularities. These results were published in 2011 in the journal *Mathematische Nachrichten*.

I have only described parts of Sławomir Rams's work, but these parts are in my opinion already sufficient to show that Sławomir Rams has been able to attack and make progress on difficult problems in algebraic geometry. He clearly masters the necessary techniques (both classical and more recent) and geometric intuition. He has written (according to MathSciNet) 28 articles, published in good, and sometimes very good, journals. He also likes to interact with other people (all of his publications since his habilitation thesis are written with collaborators), which is always an asset in a department. He has also already supervised successfully two PhD theses and is starting with a third student.

In my opinion, Sławomir Rams has all the necessary qualities to become a University professor and I recommend him for the title of professor at Jagiellonian University.

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