BİLKENT UNIVERSITY
DEPARTMENT OF MATHEMATICS

Alexander DEGTYAREV
Phone: +90 [312] 290–2135
Fax: +90 [312] 290–1797
E-mail: degt@fen.bilkent.edu.tr

May 22, 2018

Prof. dr hab. Włodzimierz Zwonek, Dean
Wydział Matematyki i Informatyki
Uniwersytet Jagielloński w Krakowie

Dear Professor Zwonek,

This letter is to support dr hab. **Sławomir Rams'** application for the conferment of the title of **Professor** in the field of science in mathematics.

1. Research. Professionally, I am better familiar with Sławomir's research. Therefore, I will mainly concentrate on this aspect of his work; other aspects of his scientific activity will be briefly touched upon later.

Listed in `MathSciNet` are 28 papers authored by Sławomir. However, considering his productivity and, alas, the increasingly slow publication process in the "good" journals (which Sławomir's work definitely deserves), one should add to this number at least half a dozen of papers still waiting to be printed. (Some of them, posted in the `arXiv`, are included into my report.) Major works can roughly be divided into the five groups below, dealing with quite different aspects of algebraic and analytic geometry and demonstrating the diversity of the applicant's scientific interests.

1.1. *Lines on quartics* (mainly in collaboration with M. Schütt). One of the most recent achievements by the applicant and, probably, the most influential part of his work is a large series of papers (I found nine, but I presume that this work is still in progress) on the maximal number of lines in spatial surfaces, most notably quartics. (Several papers have already been published in leading journals, and some are still in the refereeing process.) This is a beautiful classical algebra-geometric problem that goes back as far as to F. Schur (1882), who constructed a smooth quartic with 64 lines. After a number of fruitless efforts, the problem for quartics was seemingly settled by B. Segre (1943), who proved that 64 is indeed the maximum. However, Rams and Schütt (2015) discovered a serious flaw in Segre's arguments, which were based upon the erroneous claim that a line may meet at most 18 others. The true bound of 20 other lines is attained in a certain explicit family of quartics, discovered by Rams and Schütt and subsequently studied by many other authors, which also plays an essential rôle in a number of other line related problems. However, in spite of this new discovery, in the same paper Rams and Schütt managed to bridge the gap and prove that 64 is still the maximum. Furthermore, employing purely geometric arguments, they extended this bound from complex quartics to all fields of characteristics other than 2 or 3. In two subsequent papers, the remaining cases of characteristic 2 and 3 have also been settled, with the sharp bounds of 60 and 112 lines, respectively.

This work (including a number of auxiliary papers of more technical nature) gave burst to a new interest to this old classical problem, making it a popular research area. The original problem can be generalized to singular quartics (an upper bound of 48 lines in the presence of a non-simple singular point was obtained by S. Rams and V. González-Alonso), supersingular quartics over fields of positive characteristic, $K3$-surfaces with other polarizations, and surfaces of other degrees. Some problems have been settled, by a number of researchers including the applicant himself, whereas many others are still wide open. For example, in a forthcoming paper by Rams and Schütt, Segre's bound of 147 lines for a smooth quintic is significantly improved down to 127 (which is probably still not sharp, the current champions possessing but 75 lines). The original techniques used by the authors, including improved bounds on the valency of a line, may shed new light to the line counting problem in higher degrees, where the known upper and lower bounds are still quite different (approximately $11d^2$ vs. $3d^2$ lines realized by the Fermat surface of degree d). Yet another refinement of the problem would be the classification of large configurations of lines (another vast area of research) and study of various special configurations; I would presume that Sławomir's interest in the line problem was partially motivated by his earlier papers on surfaces with many *skew* lines, showing, in particular, the asymptotic sharpness of Miyaoka's bound.

1.2. *Three-divisible divisors.* Another impressive series of four papers (two of which are in collaboration with W. Barth) is the study of the so-called 3-*divisible* divisors on various surfaces. Roughly, this notion generalizes Zariski's sextics of torus type (or rather pull-backs of their cusps in the respective double planes), so that the surface admits a cyclic triple cover ramified at the divisor. Starting with 3-divisible configurations of skew lines (the first paper), the authors go on and consider configurations of (the exceptional divisors of the) cusps on spatial surfaces, giving a detailed description of such configurations and underlying surfaces in small degrees (at most six) and providing a plethora of examples in higher degrees. Note that cusps are simplest singularities for which 3- (and only 3-) divisibility makes sense, as their links are the lens spaces of type $(3, 1)$, the fundamental groups being of order 3. Naturally, cuspidal quartic surfaces are quite similar to double planes ramified at cuspidal sextic curves (both are $K3$-surfaces), and this similarity extends to the configurations of cusps: any divisible configuration consists of 6 cusps, any configuration of more than 6 cusps contains a 3-divisible one, etc. (This is one of the first results obtained by the applicant in this direction.) Curiously, the geometry of 3-divisible sets of cusps changes when the degree grows: thus, although there are no direct homological obstructions, a 3-divisible set in a sextic surface would contain at least 18 cusps!

It is expected that a further understanding of the 3-divisible sets will lead to a sharp bound on the maximal number of cusps in a surface of a given degree. As another by-product, the authors use 3-divisible sets of cusps or those of skew lines to construct new linear ternary codes with "good" parameters. (In the $K3$-case, one can start with an appropriate ternary code and, thanks to the surjectivity of the period map, prove the existence of the corresponding cuspidal surface by purely lattice theoretical means. However, the authors choose the opposite approach, which also works in higher degrees, deriving interesting codes from explicit defining equations.) One may anticipate further applications of this work in coding theory and cryptography.

1.3. *Quasiadjunction type formulas* (partially in collaboration with S. Cynk). These three papers follow the ideas of Zariski (developed later by several other authors) and compute the Hodge numbers of (ramified) cyclic coverings of singular varieties (or those with singular ramification loci) in terms of the (twisted) sheaves of logarithmic forms. In short, Deligne's sheaves giving rise to the mixed Hodge structure in the homology of the covering are projected downstairs and broken into eigensheaves. These eigensheaves are not quite locally free: at the singular points they are to be tensored by the so-called *quasiadjunction ideals*; in other words, singularities of the forms are to be refined in a certain way. At the end, one expresses the Hodge numbers of the coverings in terms of the twisted Hodge numbers of the base, dimensions of the quasiadjunction ideals (which depend on the types of the singularities only), and certain *defects* that are very similar to the classical superabundance (difference between the actual and virtual dimensions) of linear systems.

1.4. *$K3$- and related surfaces.* In addition to those mentioned above (lines on quartics and cuspidal quartics), there are a few other papers on $K3$- and related (most notably Enriques) surfaces. These include a study of automorphisms of Enriques surfaces and their entropy, classification of cuspidal Enriques surfaces, a description of the big cone of an Enriques surface, "strong" (jet related) ampleness of line bundles on $K3$-surfaces, etc. It is worth mentioning that, instead of the traditional non-constructive lattice theoretical methods, the authors treat $K3$-surfaces from a more geometric viewpoint. This approach gives one finer control over the subtle geometric properties and lets one extend the results to fields of positive characteristic and/or wider classes of varieties; in some cases, the authors derive their assertions for $K3$- and Enriques surfaces from far more general statements, quite interesting in their own right.

1.5. *Intersections of analytic sets.* These earlier papers were published before or right after Sławomir's graduation in 1999, so I presume that they constitute and extend the contents of his Ph.D. thesis. Here, the author extends the intersection theory to certain analytic sets (so-called *positive holomorphic chains*). This extension requires a detailed study of the topology of the space of such sets and their deformations. Alternatively, in a couple of later papers, the intersection indices are also defined algebraically, in terms of the local rings of the sets. One of the principal achievements of this work is a Bézout type theorem for analytic sets.

1.6. *Miscellanea.* Sławomir has also authored, alone or in collaboration, half a dozen of papers on a number of other diverse algebra-geometric problems.

Particularly worth mentioning are several works on higher dimensional varieties, e.g., a detailed study of the geometry of the Coble–Dolgachev sextic or various bounds on the number of prescribed isolated singular points in a threefold of a certain kind (e.g., nodes in non-factorial complete intersections or simple triple points in Calabi–Yau quintics).

Two papers deal with linear systems of quadrics: in addition to the geometric beauty of this subject, it has many applications to semidefinite programming. To any such system, one can associate two varieties, viz. the base locus and the double space ramified at the discriminant locus; in certain dimensions (pencils in $\mathbb{P}^3$, nets in $\mathbb{P}^5$, webs in $\mathbb{P}^7$, etc.) both are Calabi–Yau manifolds that, under mild conditions, are expected to be birationally equivalent to each other. The former case (elliptic curves) is classically well known; in the two latter ($K3$-surfaces and Calabi–Yau threefolds), the precise conditions are established and explicit birational maps are described by S. Rams and S. Cynk.

Overall, I am impressed by the variety of subjects addressed in Sławomir's work and the diversity of tools used, from algebraic geometry to arithmetic and analysis, clearly demonstrating the rich mathematical background of the applicant.

2. Didactic activity. To my knowledge, Sławomir has taught a great variety of mathematical courses, at all levels, both in Poland and in Germany (Gottfried Wilhelm Leibniz Universität Hannover), acquiring the necessary teaching experience.

Sławomir has supervised three Ph.D. students. One of them, Anna Antoniewicz, has already graduated (2011) and published a paper, whereas the two others are still working towards their degrees. As far as I know, unofficially Sławomir has taken active part in guiding M. Schütt's student Davide Veniani, and the latter has warmly acknowledged Sławomir's contribution. Currently, D. Veniani is continuing successful scientific career at Johannes Gutenberg-Universität Mainz; in spite of his young age, he has authored six paper, three of which have already been published.

3. Conclusion. Summarizing, I conclude that dr hab. **Sławomir Rams** is a gifted mature mathematician who has already made quite considerable contribution to the field of algebraic geometry and is still continuing active and important research activity. Sławomir is a perfect scientific leader; he has established strong ties with a number of national and international teams on a wide variety of subjects; many of his joint projects stem from his original papers and are fueled by his ideas. Several papers authored by Sławomir turned out quite influential, attracting wide attention and opening new research areas.

Sławomir is also an excellent teacher and a knowledgeable and patient supervisor; I think that he is ready to found and lead a local scientific school.

Therefore, I strongly support the application and recommend the bestowing of the title of Professor in the field of science in mathematics upon dr hab. Sławomir Rams.

Yours sincerely,

Alex Degtyarev