REPORT ON THE Ph.D THESIS

Evolutionary Multivalued Hemivariational Inequalities Modeling Dynamic Unilateral Contact Problems

by

Mr. JIANGFENG HAN

The thesis of Mr. Jiangfeng Han deals with the study of a mathematical model which describes the contact of a viscoelastic body with a deformable foundation. This topic is very important since such kind of problems arises in civil engineering and industry, especially in automotive industry and in metal forming. Therefore, there is a real need to provide a reliable analysis of the corresponding problems, both from the theoretical and the numerical point of view.

Contents. The thesis is structured in seven chapters and a reference section. A brief description of the contents of each chapter is the following.

Chapter 1 represents an introductive one. Here, the author underlines the importance of variational and hemivariational inequalities in both Nonlinear Analysis and Contact Mechanics and provides basic references in the field. Then he proceeds with a description of the contents of each chapter of the manuscript.

Chapter 2 gathers most of the preliminary material needed in the next chapters of the manuscript. It contains background on spaces of vector-valued functions, basic properties of various classes of single-valued and multivalued nonlinear operators of monotone type on Banach spaces, as well as basic definitions and properties of the subdifferential mapping, both in the sense of the convex analysis and in the sense of Clarke.

Chapter 3 introduces the model of contact whose analysis represents the main aim of this thesis. The model is constructed assuming that the contact process is dynamic and the material's behaviour is viscoelastic. The later is described with a version of the Kelvin-Voigt constitutive law in which the viscosity operator is nonlinear and time-dependent while the elasticity operator is linear. The contact is described with a normal damped response condition associated to unilateral constraints in velocity. Finally, the friction is described with a slip-rate dependent version of Coulomb's law of dry friction. Considering such kind of frictional contact conditions represents the first trait of novelty of the manuscript and leads to a new an interesting mathematical model (Problem 3.1). Next, the author states the assumptions on the problem's data and derive a variational formulation of the problem which

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is in a form of an evolutionary multivalued hemivariational inequality for the displacement field (Problem 3.2.).

Chapter 4 represents one of the main chapters of the manuscript. Here, the main results are Theorems 4.1 and 4.2 which state the solvability and the unique solvability of the contact problem, respectively. The main idea in the proofs of these theorems is to reformulate Problem 3.2 as an evolutionary inclusion (Problem 4.1) and to prove its solvability by using a surjectivity result for multivalued pseudomonotone operators. The uniqueness of the solution of Problem 4.1 also follows, under a smallness assumption on the data. Based on these intermediate results the author concludes the proof of Theorems 4.1 and 4.2. In the proofs presented in this chapter he uses nice and interesting arguments of functional analysis and measure theory, providing a deep knowledge of these mathematical tools.

Chapter 5 concerns the study of the optimal control of the frictional contact problem. Three kind of problems are considered: the optimal control problem via external forces and initial conditions, the time optimal control problem, and the maximum stay control problem. For all these problems the existence of an optimal solution is proved (Theorems 5.1-5.3). The proofs are based on arguments of compactness and lower semicontinuity.

Chapter 6 deals with the study of spatially semidiscrete approximation of Problem 4.1. Here, the author introduces a discrete approximation of the problem (Problem 6.1) for which he proves the existence of a unique solution. Then he derives an error estimate result (Theorem 6.1) and, under additional solution regularity, he obtains an optimal order error estimate result (Corollary 6.1).

Chapter 7 consists in a short conclusion and a description of some problems for further research. They include the study of fully discrete schemes for the contact problem (left open in this manuscript) and the study of more complicate dynamic problems in which additional effects like wear, damage, adhesion and thermal effects are included.

Evaluation. The thesis of Jiangfeng Han is a welcomed contribution to the *Mathematical Theory of Contact Mechanics*. It contains relevant and interesting results in the study of a new dynamic frictional contact model. The novelty of the model arises in the boundary conditions the author considers, which involves unilateral constraints, strong nonlinearities and time-dependence of various functions. The results obtained in the analysis of this model are new and interesting. They include: the existence and uniqueness result for the weak solutions to the contact model (Theorems 4.1 and 4.2) the existence of optimal control for three kind of problems (Theorems 5.1-5.3) and the error estimates for semidiscrete schemes (Theorem 6.1).

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The results presented by Mr. Jiangfeng Han are of very high academic quality and have the potential to be employed in engineering practice. Also, they open the way to the abstract study of new classes of multivalued hemivariational inequalities with unilateral constraints, including their optimal control and their numerical analysis.

The manuscript presented by the candidate is well structured and very well written. It proves that the author has very thorough skills of functional analysis, nonlinear analysis, convex analysis, optimal control and theoretical numerical analysis. The candidate has not only a good understanding of various functional analysis techniques, but he is also able to apply them in the study of new and interesting mechanical problems. His theoretical results are original and of substance. Also, the list of the references presented at the end of this manuscript is complete and includes relevant references in the field.

Conclusion. In conclusion, based on these facts, I consider that Mr. Jiangfeng Han is an accomplished applied mathematician which well deserves the degree *of Ph.D of the Jagiellonian University* with honorable distinction. I support his application in very strong terms.

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