## Abstract for the PhD thesis *Rigorous numerical computation of the Conley index for flows* by Frank Weilandt, UJ KMO

This thesis deals with the numerical computation of the Conley index, a homological invariant defined for two types of dynamical systems: flows $\varphi : X \times \mathbb{R} \to X$ and continuous maps $f : X \to X$ on some Hausdorff space $X$. The Conley index measures the behavior of the system around an invariant set $S$ if it has a neighborhood $M$ such that $S$ is the invariant part of $M$. The Conley index depends only on $S$ and not on $M$, but $M$ can be represented more easily on a computer.

We numerically compute the Conley index for a Poincaré map $P : \mathbb{R}^d \to \mathbb{R}^d$ coming from a non-autonomous ordinary differential equation $\dot{x} = v(t,x)$, $v : \mathbb{R}^{1+d} \to \mathbb{R}^d$, which is periodic, i.e., there is a number $T > 0$ such that $v(t+T,x) = v(t,x)$ for all $(t,x)$. One standard idea would be the numerical enclosure of the image of small boxes under $P$ after time $T$. This is infeasible if the solution curves expand very quickly. Another difficulty is that $P$ is often not defined everywhere in $\mathbb{R}^d$, but only on a subset.

We present a theorem to deal with this situation, which allows us to compute the Conley index of $P$ integrating the system (the flow in the extended phase space $\mathbb{R}^{1+d}$) for a time step much smaller than the return time $T$ of the Poincaré map. We sketch its proof and present an algorithm checking the prerequisites of the theorem numerically. The algorithm together with a proof of its correctness constitutes the main content of this thesis. We also give example outputs from the author's implementation.

Additionally, we are interested in computing Morse decompositions for flows. In the classical approach, one would analyze the map $\varphi_h(x) = \varphi(x,h)$ for some given $h > 0$ using existing software for maps $f = \varphi_h$. But there is no reasonable heuristic for choosing $h$. We present a more flexible strategy in which we let $\tau : X \to (0,\infty)$ be any continuous function and $f(x) := \varphi(x,\tau(x))$ the map which we analyze numerically using existing tools. This allows us to propose a heuristic for choosing $\tau(x)$ depending on the norm of the given vector field at $x$.

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