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Report on the Habiliation Thesis of Krzysztof Turowski

The habiliation thesis of Krzysztof Turowski consists of 5 parts:

[A1] Krzysztof Turowski, Wojciech Szpankowski, Towards Degree Distribution of a Duplication-Divergence Graph Model, The Electronic Journal of Combinatorics, 28(1) (2021), P1.18.

[A2] Alan Frieze, Krzysztof Turowski, Wojciech Szpankowski, Degree Distribution for Duplication-Divergence Graphs: Large Deviations, 46th International Workshop on Graph-Theoretic Concepts in Computer Science, WG 2020, Leeds, UK, June 24-26, 2020. Lecture Notes in Computer Science 12301, pages 226-237.

[A3] Alan Frieze, Krzysztof Turowski, Wojciech Szpankowski, The concentration of the maximum degree in the duplication-divergence models, Proceedings of 27th International Conference of Computing and Combinatorics, COCOON 2021, Tainan, Taiwan, October 24-26, 2021. Lecture Notes in Computer Science 13025, pages 413-424.

[A4] Philippe Jacquet, Krzysztof Turowski, Wojciech Szpankowski, Power-Law Degree Distribution in the Connected Component of a Duplication Graph, 31st International Conference on Probabilistic, Combinatorial and Asymptotic Methods for the Analysis of Algorithms, AofA 2020, June 15-19, 2020, Klagenfurt, Austria (Virtual Conference). LIPIcs 159, pages 16:1-16:14.

[A5] Krzysztof Turowski, Abram Magner, Wojciech Szpankowski, Compression of Dynamic Graphs Generated by a Duplication Model, Algorithmica 82(9) (2020), pages 2687-2707.

All papers deal with special dynamical random graph models, the duplication model in [A4] and [A5] and the duplication-divergence model in [A1]-[A3].

The theory of random graphs goes back to the seminal work of Erdös and Renyi who introduced the G(n,p) model and observed already various properties (most importantly the birth of the giant component when p passes the threshold 1/n). Since then the theory of random graphs developed rapidly and is still a very active and broad field of research with applications from physics to biology and computer science. In particular various new models have been introduced that capture the dynamics of “real world networks” like proteins or the world wide web.

The duplication-divergence model DD(t, p, r) – that is studied in [A1]-[A3] – was introduced by Solé, Pastor-Satorras, Smith, and Kepler in 2002. Starting from a given graph on t0 vertices (labeled from 1 to t0) new vertices labeled by t0 + 1, t0 + 2, …, t are added subsequently as copies of some existing vertices in the graph. At each step some edges connected to the new vertex (with label s) are removed with probability p or new ones are added with probability r/s. Finally all labels are removed. In the duplication model that is used in [A4] and [A5] the addition part is left out. In both cases these models capture several properties from real world networks like biological processes in protein interaction networks (and others).

The main focus of the papers [A1]-[A4] is the asymptotic study of the degree distribution that depends heavily on the input parameters, in particular there is phase transition in the behavior if p = ½. Paper [A5] deals with compression problems for the duplication model.

In the first paper [A1] the authors study the average degree over all vertices in DD(t, p, r) as well as the average degree of the s-th vertex in the generation (with s between t0 and t) and provide very precise asymptotic results for all parameter values p. As mentioned above the cases 0< p < ½, p = ½, and ½ < p < 1 lead to different asymptotic results with a phase transition at p = ½. Furthermore the growth order of the variance is given, too. The analysis uses the property of the underlying Markov process in order to obtain proper recurrences that are then asymptotically analyzed.

In the second paper [A2] these results are complemented with corresponding large deviation results for the average degree and for the degree of the s-th vertex in the generation. Again the Markov property is used, however, now in order to obtain bounds for the moment generating function so that Chernoff’s bound can be applied in order to obtain large deviation bounds.

The third paper [A3] focuses on the maximum degree that is an important parameter in random graph theory. The main result says that with high probability the maximum degree (for ½ < p < 1) is (more or less) concentrated around t^p. The proof is actually very tricky and technical (and by no means obvious). Again Chernoff-type bounds play an important role.

The last two papers [A4] and [A5] consider the pure duplication model, where the addition part is left out (that is, r=0).

The fourth paper [A4] is a very interesting one. It rigorously establishes an approximate power law for the degree tail distribution for 0 < p < 1/e that is of the form C/k^beta, where beta satisfies the equation p^(beta-2) + beta-3 = 0. The analysis makes use of several sophisticated analytic techniques like the Mellin transform.

Finally the fifth paper [A5] deals with a completely different question, namely how such random graphs can be optimally compressed. It turns out that the labeled version needs O(n) bits whereas the unlabeled version just need O(log n) bits due to the significant amount of symmetry.

All works are characterized by great originality and technical mastery. Overall, they represent a substantial scientific advance in the field of random graphs. Furthermore, all parts of the habilitation thesis are carefully and clearly presented. And the results are of high mathematical quality.

Summing up, I am convinced that Krzysztof Turowski is an outstanding candidate for the habilation at the Jagiellonian University and therefore I strongly support his application.

Sincerely, yours

Michael Drmota
Professor for Discrete Mathematics