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Report on the doctoral dissertation

Selected Risk-sensitive Optimal Stopping and Impulse Control Problems

by Damian Jelito

submitted for the degree of Doctor of Philosophy

The Ph.D. thesis by Damian Jelito is devoted to the risk-sensitive optimisation problems, that is, various problems where the optimality criterion is based on the utility function of a specific form with a parameter γ representing the risk-aversion of an agent. It should be acknowledged that the existing literature pertaining to risk-sensitive optimisation problems is vast both in the discrete and continuous time setting with over 100 papers given in the comprehensive list of references and thus it was a challenging task to furnish original contributions to the well-established field.

In his doctoral dissertation, the author concentrates on two particular problems, namely, the optimal stopping problem studied in Chapter 3 and the impulse control problem examined in Chapter 4. His choice is motivated by the desire to obtain results that can be implemented in practice and thus make them useful in the context of real-world applications of stochastic control theory. Therefore, in Chapter 5 he analyses practical aspects of results from Chapters 3 and 4 with special attention paid to verifications of conditions appearing in various theorems in some instances of stochastic models. However, it is fair to observe that even in Chapter 5 the approach is definitely theoretical and focused on finding explicit solutions for some toy examples, rather than numerical methods for complex problems.

The thesis is very well written, gives a fairly comprehensive analysis of theoretical methods and their potential applications, and proposes several original extensions of results known from the existing literature. The technical quality of his original contributions and the style of the thesis are excellent. The obtained results are suitable for publication in research journals on stochastic processes and optimal control and, in fact, three joint papers with his supervisors have already been published in leading international journals: *SIAM Journal on Control and Optimization, Stochastic Processes and their Applications* and *Electronic Journal on Probability.*

In Chapter 2, the author introduces the setup and gives some definitions and properties that are needed in the foregoing chapters.

The main original results on the risk-sensitive optimal stopping with infinite and finite time horizon in a Markovian setup are formulated and proven in Chapter 3. The authors examines there two cases, namely, when stopping times are arbitrary and when the maximisation problem is restricted to the class of all bounded stopping times.

The principal idea in the proofs of main results hinges on showing that in both cases the corresponding value function can be characterised as a solution to a suitable Bellman equation. Under several technical assumptions, which are mainly related to the boundedness of the terminal reward, the author first derives the discrete time version of the Bellman equation and subsequently studies in Theorem 3.2.6 solutions to the above-mentioned optimal stopping problems. In the next step, in order to provide a link between the discrete and continuous time setups, a continuous time Markov process is studied but optimal stopping problems are examined under the postulate that stopping times take values in a discrete set. Subsequently, the main problem in continuous time with finite time horizon is studied for time-homogeneous and time-inhomogeneous case. The proofs of the respective results, Theorems 3.3.2 and 3.3.4, are quite technical but all details of both proofs are provided. Using similar techniques, the case of infinite time horizon is addressed and solved in Theorem 3.4.7. The chapter concludes by an analysis of approximation schemes for risk-sensitive optimal stopping problems.

Chapter 4 is devoted to a completely different optimisation problem, namely, the risk-sensitive impulse problems. Similarly to Chapter 3, the finite horizon problems long-run problems are considered and the main technique is the derivation of the Bellman equation followed by a suitable verification theorem, Theorem 4.2.2 for the finite horizon case and Theorem 4.3.4 for the long-run case. Subsequently, the existence of a solution to the respective Bellman equation is studied in Theorems 4.2.5 and 4.5.8. As was already mentioned, in Chapter 5 the author considers more specific classes of Markov processes and shows that technical conditions from previous chapters can be verified. It is also emphasised that the main results from the thesis cannot be easily generalised since the issue of non-uniqueness of solutions to the Bellman equation may arise if some conditions are relaxed. In particular, in Examples 5.3.1 and 5.3.5 two highly non-trivial instances of non-uniqueness of a solution to the Bellman equation are very carefully analyzed. Finally, some auxiliary results from the literature are collected in the appendix.

Summary. It was my pleasure to be an examiner for the thesis of Damian Jelito since I have greatly appreciated both the mathematical content and very careful presentation of results and proofs. The author has shown his familiarity with the existing literature, a deep understanding of results obtained in other works and a detailed knowledge of mathematical techniques and fine arguments used in proofs. He has established several original results, especially in Chapters 3–5, which were already published in leading journals. I have found the research by Damian Jelito very interesting and well carried out. The thesis is written in clear, mathematical style, more suitable for academics than to practitioners, but very understandable nonetheless. One would wish that this line of research continues to attract the attention of other young researchers and that our understanding of risk-sensitive optimisation problems continues to grow, thanks to works like the present one. It is clear that the results derived in the thesis will provide foundations for further practical modelling work in this area.

Conclusions. The thesis is clearly written and represents a significant contribution to the field of risk-sensitive optimal stochastic control. The research of Damian Jelito presented in his doctoral dissertation is valuable from the mathematical viewpoint and it may also bring essential contributions to practical issues arising in the context of risk-sensitive optimisation problems.

I am convinced that Damian Jelito should be awarded the degree of Doctor of Philosophy for his dissertation *Selected Risk-sensitive Optimal Stopping and Impulse Control Problems* and its my pleasure to recommend award of the degree of Doctor of Philosophy with a special mention.

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M. Rutkowski