

JAGIELLONIAN UNIVERSITY

*Abstract*

Institute of Mathematics of the Jagiellonian University

Doctor of Philosophy

**Integrability, Foliations and Twistors**

by Rouzbeh MOHSENI

Roger Penrose introduced twistor theory and originally twistor theory associated a complex twistor space  $\mathbb{C}^4$  (or sometimes its projectivised twistor space  $\mathbb{CP}^3$ ) to the space of light rays in space-time. Although twistor theory was formulated to provide a new path to quantum gravity, that is not yet complete due to many technical issues it has faced, it has several other applications in mathematics. One of the applications is in the study of harmonic maps and in this case the twistor space is the space of all almost complex structures on an even-dimensional manifold.

We develop the theory of twistor spaces on foliated manifolds and construct the twistor space of the normal bundle. We demonstrate how the classical constructions of the twistor theory lead to foliated objects and permit to formulate and prove foliated versions of some well-known results on harmonic mappings. Then since any orbifold can be realized as the leaf space of a suitable defined Riemannian foliation, we obtain orbifold versions of the classical results as a simple consequence of the results on foliated mappings.

We use the algebraic techniques of Picard-Vessiot theory to shed some light on the integrability of real dynamical systems. In particular we provide an example and prove that it is not integrable, however, it possesses tame topology. This shows the assumption that non-integrability is equal to chaos is absolutely invalid. Then we provide some more examples of real dynamical systems and some insight on the Arnold-Thom conjecture.

*Rouzbeh Mohseni*