Institute of Mathematics
Polish Academy of Sciences

Warszawa 15.03.2022

Referee report of the PhD thesis

by mgr Szymon Myga

entitled

"Convex and symplectic properties of Monge-Ampére - type operators on compact Kähler manifolds with holomorphic torus action"

The thesis under consideration deals with the fundamental in complex analysis and differential geometry complex Monge-Ampére type operators, or complex Monge-Ampére measures on compact Kähler manifolds. The operators (or Monge-Ampére equations) expressed by Hessian determinant of the holomorphic and antiholomorphic derivatives $\partial\bar{\partial}$ have very important applications in several mainstream directions of complex geometry and mathematical physics. Moreover, enriched with the Hamiltonian torus action on the respective Kähler manifold brings the new and interesting results concerning volume forms on Kähler manifold and symplectically reduced Kähler orbifolds. Author concentrates on the structural connection between real and complex features of the Monge-Ampére operators and gets important results in an advanced, relatively young and intensively expanding domain of pluripotential theory and harmonic analysis on compact Kähler manifolds.

In the first, introducing part covered by sections 2-5 author clearly formulates

the most important subjects of the necessary background material. The elements of convex analysis with the basis of Monge-Ampére equation, measure theory and optimal transport. Complex geometry, Kähler manifolds, their holomorphic vector bundles and first steps of symplectic geometry. The last two sections of introduction include the pluripotential theory on compact Kähler manifolds, in particular very fundamental notions of this theory like currents, pluri-fine topologies, Monge-Ampére measure and energy - weighted energy for locally bounded plurisubharmonic functions. The Lie group actions on manifolds with the emphasize of Hamiltonian torus actions on Kähler manifolds is extensively presented in section 5 with the detailed description of momentum maps and Marsden-Weinstein reduction and getting partial original results around Borel construction and the orbifold properties.

An essential part of the thesis is contained in sections 6-8. The several important results obtained in toric pluripotential theory for Kähler toric varieties, among them: exact formulas for the momentum map on comlex tori and the reduced form on the regular level of momentum map by Marsden-Weinstein reduction, through the celebrated cohomological equation (Theorem 6.10) and by handy properties of Legendre transform there was obtained an important relation between the Kähler volume and volume of the reduced space (Theorems 6.12, 6.14). The new g-Monge-Ampére measure was constructed in section 7 with the envelope operator included in the building block of the Monge-Ampére operator ($MA_g(\phi)$). The exact construction and the continuity property of the Monge-Ampére operator were carefully proved in several preparatory results (Lemmas 7.2, 7.3, Proposition 7.4) and continuity theorem (Theorem 7.5) in T-fine topology. The g-Monge-Ampére equation of the form, $MA_g(\phi) = \mu$ in its generality and the whole variety of motivations was presented in section 8. First author deals with case of the equation on toric varieties with Monge-Ampére measure on $\mathbb{R}^n$ and probability measure on the moment polytope and proves existence of the solution of the Monge-Ampére equation, so called dual Brenier solution (Theorem 8.1), which

in fact induces a unique solution on compact toric varieties. The existence of solutions of g-Monge-Ampére equation on general manifolds under certain weak conditions was proved in Theorem 8.7. but the uniqueness of solutions was proved on compact Kähler orbifolds (Theorem 8.10 and proposition 8.11.). The proofs applied use the advanced methods of complex analysis, complex optimal transport and pluripotential theory in general.

The paper is well written without major errors. Author succeded very well introducing properly many difficult notions of analysis and geometry. To each resulting subject his introduction is necessary and complete. The results are based on the nontrivial achievements of leading experts of the field and the proofs of his results are smooth and clear. I would emphasize the results for Monge-Ampére objects especially analysed on toric varieites with its extra structure. In conclusion I would like to say that the PhD thesis of Szymon Myga contains the new results giving an important contribution to the theory of Monge-Ampére operators on compact Kähler manifolds and fulfills the requirements of the law in this matter. No doubt he should be allowed to the next stages of the doctoral procedure.

prof. dr hab. Stanisław Janeczko