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Assessment of PhD Desertation "Metody teorii indeksu Conleya w dynamice próbowej" by Mateusz Przybylski

The thesis takes the subject of dynamical systems under scrutiny. On the one hand, it is a very classical research subject; on the other, qualitative analysis of dynamical systems is a very vivid research subject. In particular topological methods, specifically the Conley index theory, for dynamical systems is a promising subject of potential in numerical methods. Since the methods discussed and developed in the thesis can work with time series, a connection with machine learning is also apparent.

The manuscript consists of 7 chapters:

Chapter 1 (Wprowadzenie): The first chapter gives a brief but sufficient background to topology, category and dynamical systems. It is used to establish notation and style throughout the thesis.

Chapter 2 (Recontrukcja przestrzeni I uklady dynamicznego): This chapter deal with a discre dynamical system which values are sets - multivalued dynamical systems. Here, the notion of delta-cube (kostka) and cubical and sunflower enclosers are introduced and discussed.

Chapter 3 (Indeks Conleya): This chapter gives a careful introduction to Alexander-Spanier's cohomology. Subsequently, the Conley index is introduced for a multi-valued dynamical system. The examples used in this section illuminate the concepts. Chapter 2 and 3 comprise a general introduction to Conley index theory and its modifications.

Chapter 4 (Istnienie orbit) leans upon [3] and [4], which are the original works of Przybylski together with his PhD supervisors Bratko and Mrozek. Here, the reader is introduced to the topological condition under which an orbit of a dynamical system exists. The results of this section are used to derive abstract algorithms which verify that the index map satisfies Lefschetz completeness.

Chapter 5 (Obliczenia indeksu Conleya) is the one that combines the results from the previous four sections and provides a consistent method of computing Conley index. The challenge here is to convert the theory developed for multivalued maps to the existing software packages.

Chapter 6 (Wyniki obliczniowe) provides examples. Firstly, a numerical experiment with the two-dimensional Hénon map then three-dimensional counterpart are carried. Finally, a physical experiment is executed, and analysis is carried out on the time series of measured values from an experiment. All three experiments verify the applicability of the derived methods in the practical context.

Chapter 7 (W kierunku indeksu Conleya dla relacji binarnych) comprises a first step towards a new definition of Conley index for discrete dynamical system using Szymczak's category.

In conclusion, the thesis deals with the important subject of topological methods for characterising the behaviour of dynamical systems. The work is meticulous concerning the notation, definitions, theorems and their proofs. It has a good, gentle introduction to the subject. Subsequently, the main results show how to compute the Conley index and apply it in numerical and physical experiments. The thesis is on a high scientific level. Therefore, I conclude that the thesis fulfils the conditions specified in the Act of March 14, 2003 on academic degrees (t.j. Dz. U. 2017, poz. 1789).

Sincerely yours

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