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Report on the professional achievements of Artur Piękosz

The roots of Artur Piękosz’s work are located in important areas of pure mathematics, such as o-minimality and the work of Delfs and Knebusch on the construction of semi-algebraic versions of homology, cohomology and homotopy theories. In particular, research in o-minimality is still flourishing today and producing significant results. Generally stated, Piękosz’s aim is the generalization of central results and techniques from these areas, inspired by Knebusch’s suggestion that a further generalization of the theories to the case of suitable o-minimal structures should be considered. Piękosz is motivated by the observation that other authors so far had only generalized small subsets of Delfs’ and Knebusch’s results. So he fills the gap in the extensive article [H.1], working with any o-minimal expansions of fields, not only of the reals.

Piękosz also observes that in the work of the other authors the importance of the underlying generalized topological spaces (a notion introduced by Delfs and Knebusch) had not been clearly worked out. This leads him to a thorough investigation of generalized topological spaces, undertaken in the papers [H.2]–[H.5]. In [H.2] he presents an improved axiomatization of generalized topological spaces. Then in both [H.2] and [H.3] a considerable number of generalized topological concepts and categories related to the category **GTS** of generalized topological spaces are introduced and studied. Joining forces with Eliza Wajch, notions of compactification of generalized topological spaces are studied in [H.4], and in [H.5] the connection with bornological universes is worked out and generalized notions of metrizability are considered. The collaboration with Wajch adds the additional feature that most work is done in ZF without the axiom of choice, and wherever necessary, it is shown which possibly weaker axioms can be used in its place.

In the introduction of [H.2], Piękosz writes: “The author hopes that from now on generalized topology (hidden in the language of locally semialgebraic spaces of [DK], and weakly semialgebraic spaces of [K1]) will be developed without constraints.” In the papers that make up the habilitation achievement, he contributes a large body of work to this development. This fact alone speaks for the prolific nature of Piękosz’s work.

Other aspects of Piękosz’s activities which I find positive and worthwhile mentioning are:

a) Piękosz paid extended research visits to important places of mathematical research:

- (i) the University of Illinois at Urbana (which can be considered a main center for o-minimality), supported by a NATO Advanced Fellowship;
 - ii) the University of Oxford in the framework of the RAAG (Real Algebraic and Analytic Geometry) program;
 - iii) the Fields Institute for the Thematic Program on O-minimal Structures and Real Analytic Geometry.
- b) Piękosz was a member of the influential RAAG network.
 - c) Piękosz has given presentations on several international conferences
 - d) Piękosz has supervised a satisfactory number of Bachelor and Master's students.
 - e) Piękosz wrote lecture notes for four different courses that were published by his university.

In my view, the above points demonstrate a well-rounded profile in research and teaching. Therefore, I support Piękosz's application for the title of habilitation.

This being said, I believe a number of critical remarks are in order.

A) In my opinion, the Self Report is not well written, and I find it rather unpleasant to read. I am missing the thread that carries the reader through a large number of definitions and results, none of which are clearly marked as central or most important. Less could have been more. The problem is aggravated by the fact that the Self Report is not a searchable pdf file.

Moreover, the exposition is at times confusing. One example is the use of the notion "generalized topology". Early on, Császár's definition is introduced (generalized topology = collection of sets closed under unions). The way Piękosz presents it, it reads like a general definition for the entire Self Report. Later, when the "generalized topological spaces" are introduced, it is not made clear that in general they are not matching Császár's definition (or more precisely, if I understand correctly, they also match Császár's definition if and only if they are partially topological gtses). Actually I do not see the reason to introduce Császár's definition at all; it would have sufficed to mention that there are definitions that are incompatible with the one used by Piękosz.

On pages 2 and 3, Piękosz puts together what he calls the background of the habilitation achievement. But what are the connections between the various items? For example, what have Grothendieck topologies to do with the programme of Delfs and Knebusch? I have not found an instance in the Self Report where this is clearly explained. Then on pages 3 and 4 follow short descriptions of the five papers that make up the habilitation achievement. I do not find it a good idea to give these descriptions before the necessary notions have been introduced. The readers are led to wonder: what are "gtses" and "SS"?

Occasionally, the readers are left scratching their head. Oops, are these really the (official/correct) first names of Peterzil and Starchenko? (In fact, "Kobi" is a nickname and "Sergey" is the first name of a different mathematician.) What is meant by "a very geometrical character of homotopy theory" which is lost by "other methods"? What are the "different approaches with similar onomastics"? What are the "sentences" in the paragraph on Wallman extensions (by the way,

every reader will certainly know what these extensions are)? And what is “regular research”?

Finally, in my opinion Piękosz does not quite manage to convey to the reader the flair of the notion of generalized topological spaces.

All of these shortcomings decrease the usefulness of the Self Report, particularly for readers who are not experts in the areas of Piękosz’s research.

B) So far, the impact of Piękosz’s work seems to be low, in particular in the areas from which he derives the topics of his studies (o-minimality and the programme of Delfs and Knebusch). Two of the leading researchers in these areas (who Piękosz mentions himself) told me that they have not followed his work and cannot say much about it. This is all the more surprising as Piękosz has been a member of the RAAG network and gave presentations on several meetings of this network, as well as at other international conferences (unfortunately it is not indicated in the CV which of the presentations were invited and which contributed talks). Also his long research visits to Urbana, Oxford and to the Fields Institute should have furthered the reception of his work.

Here is a list of all citations that I found on MathSciNet, other than those by Piękosz himself or by Eliza Wajch:

1) [H.1] is cited in the following papers:

Baro, Elias; Otero, Margarita: On o-minimal homotopy groups. *Q. J. Math.* **61** (2010), no. 3, 275–289,

Baro, Elias; Otero, Margarita: Locally definable homotopy. *Ann. Pure Appl. Logic* **161** (2010), no. 4, 488–503.

In both papers the citation is of the form “After a preliminary version of this paper was written, the preprint [12] by Piękosz has appeared with some related results.” No further details are given. I have heard from one of the authors that it later turned out that the results were actually not that related and the citation would not have been necessary. Further, Elias Baro was the reviewer of [H.1] for MathSciNet and chose to cite the summary of the paper instead of writing a review himself.

These are all “external” citations on MathSciNet for the publications forming the achievement.

2) [P.4] is cited in the following paper:

Bianconi, Ricardo: Undefinability results in o-minimal expansions of the real numbers. *Ann. Pure Appl. Logic* **134** (2005), no. 1, 43–51.

The citation reads: “The following conjecture is suggested by the work of Arthur Piękosz [8] and the results in this paper.” (No further details are given.)

3) [P.6] is cited in the following paper:

Peterzil, Ya’acov; Starchenko, Sergei: Complex analytic geometry and analytic-geometric categories. *J. Reine Angew. Math.* **626** (2009), 39–74.

The citation reads: “Compare the following result to Piękosz ([15]), where a similar type of theorem is proved in the real analytic setting.” (Again, no further details are given.) This citation is probably the most important, as Peterzil and

Starchenko are main researchers in the area of o-minimality, mentioned as such by Piękosz in his Self Report.

In this connection it may be illustrative to observe that the only goals stated in the Self Report and in the papers making up the habilitation achievement are the above mentioned: work on Knebusch's suggestion, the clarification of the role of generalized topological spaces in already published papers of other researchers, and a comprehensive study of generalized topological spaces. In all of these texts, Piękosz does not mention any other, and more concrete, possible applications of his results in the research of other mathematicians (except of himself and his coauthors).

Certainly, the development of an extended new theory with a significant number of results can sometimes proceed largely unnoticed by the mathematical community, resulting in citations being mainly self-citations or citations by a few collaborators. Occasionally the true impact unfolds only much later. Let us hope that it will unfold in the future for Piękosz's work.

Recommendation. In summary, while I see some flaws in Piękosz's achievements and his presentation of them, my opinion is that his achievements and academic stature are sufficient for the habilitation in accordance with the regulations in Poland – Ustawa o stopniach naukowych i tytule naukowym oraz o stopniach i tytule w zakresie sztuki (Dz.U. z 2016 r. poz. 882 z pn. zm.). I recommend that this title be awarded to him.



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