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Report on the scientific achievement presented by Dr. Anna Vallete for her habilitation procedure.

For her habilitation procedure Dr. Anna Valette has presented 7 papers, written either by herself or in collaboration, as her major scientific achievement. Dr. Valette works in Singularity Theory. Most of her research is concentrated around the behavior of complex or real polynomials at infinity and the so-called asymptotic critical values. This is on one side a classical topic, let me quote here Kahler's Jacobian Conjecture. On the other side, the research in this area is developing fast in recent years and has many applications.

Instead of describing in detail the content of these 7 papers, that is done perfectly by Dr. Valette in Summary of Professional Accomplishments, I will present my comments on each of these papers and summarize their contribution to the area of Singularity Theory.

The two papers on Jacobian Conjecture and Intersection (co)Homology, written both in collaboration, the first one with Guillaume Valette and the second one with Guillaume Valette and Nguyen Thi Bich Thuy, propose a new, very original approach to Jacobian Conjecture. Let me remind Jacobian Conjecture, formulated by O. H. Kahler in 1939, is a very curious problem, with elementary statement. Many mathematicians attempted to solve it and there are several theories built on these attempts but the problem persist in a stubborn way and, probably, its real nature has yet to be understood. Let me remind that Zbigniew Jelonek, one of Dr. Valette thesis advisors, is one of the leading researchers in this development, famous for introducing the set of critical values at infinity to this

context, called nowadays the Jelonek set. The approach of the papers [A1], [A2], based on a completion (up to quasi-isometry) of the pullback metric, though not successful for the problem itself (at least at the moment), is as I mentioned very original. It will probably be useful to study related problems. It is also related to the bi-Lipschitz geometry of singularities, a fast-developing area in recent years.

The paper on Sard Theorem for real closed fields is another example of original approach of Dr. Valette to a classical problem. It is also written in collaboration with Guillaume Valette. This paper contains, in my opinion, the right approach to Sard Theorem over an arbitrary real closed field and a result that is subtle and potentially very useful. Actually, Dr. Valette presents already some nice applications.

The very recent paper on Łojasiewicz Gradient Inequality, to appear in Proceedings of the A. M. S., is another contribution that, in future, could be a useful and often quoted source. It contains, in my opinion, a non-trivial interesting result based on a deep understanding the nature of Łojasiewicz Gradient Inequality on singular sets and at infinity. Let me remind here that the first such generalization of Łojasiewicz Gradient Inequality to o-minimal structures and to the boundary of the domain of definition was proposed many years ago by Krzysztof Kurdyka, the other thesis advisor of Dr. Valette.

Another very recent paper, written in collaboration with Beata Kocel-Cynk and Wieslaw Pawlucki, and to appear in Canadian Math. Bulletin, contains a new, geometric approach to the reparametrization problem of families of definable sets. This problem, in the semialgebraic set-up, stems from the work of Yomdin on the entropy of smooth mappings and the reparametrization results have been used recently in the work of Pila and Wilkie for instance. The new approach of the quoted paper is based on Pawlucki's work on Lipschitz cell decomposition.

The earlier paper written in collaboration with Beata Kocel-Cynk and Wieslaw Pawlucki contains an interesting observation on a definable-like behavior of the Hausdorff distance in the o-minimal structures. This paper is also based on Pawlucki's Lipschitz cell decomposition. Finally, the paper on Nash equisingularity contains a useful, but not difficult and somewhat standard observation, on generic Nash triviality of Nash mappings.

Let me now make some critical comments on the presented scientific dossier as a basis for the habilitation procedure. First we note that 5 of 7 presented papers are written in collaboration. This criticism is only partly valid. The very recent paper, still in press in Proceedings of the A. M. S., is deep and very original and is written by Anna Valette as the only author. The other criticism concerns the general structure of the presented work. Often happens that mathematicians construct a new theory and then develop it in several papers in order to apply it to old problems. Then this theory is sometimes associated to its author. This is how often standard scientific dossiers presented for a habilitation look like. This is not the case of Dr. Valette. Her dossier consists of several papers, all

contributing to Singularity Theory, that propose new solutions to problems in theories developed before by someone else. The only possible exception is, maybe, two papers on the metric approach to Jacobian Conjecture, that are very original. The credit for the approach given in these papers can be associated only to their authors, but the usefulness of this approach yet need to be confirmed. Finally, we observe that the papers of Dr. Valette are not quoted very often, her citation index in Mathematical Reviews is 24. Actually, the mostly quoted papers of Dr. Valette are not presented in her habilitation dossier. These are two papers . The first one written in collaboration with Marcin Bilski, Wojciech Kucharz, and Guillaume Valette, and published in Mathematische Zeitschrift. The second one is the 2005 paper on the geometry of Jelonek set published in Journal of Pure and Appllied Algebra. The latter one fits perfectly the area presented as the scientific achievement for the habilitation procedure. Let us note here again that, in my opinion, several recent papers of Dr. Valette, as for instance, the one to be published Proceedings of the A. M. S., the one on Sard Theorem, and the one on reparametrization of a family of definable sets, contain many non-trivial and subtle results that, very likely, will often be used and quoted by many mathematicians in future.

Conclusion. The scientific achievements presented by Dr. Valette for her habilitation procedure contain several important and very original results that contributed significantly to the study of the asymptotic values at infinity of complex or real polynomials. This is fast-developing and important area of Singularity Theory and Dr. Anna Valette is participating actively in this development. Therefore, I am favorable to award to Dr. Anna Valette the degree of "doktor habilitowany".

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