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Report on the doctoral thesis *Constrained Variational and Hemivariational Inequalities with Applications* by Shengda Zeng from Jagiellonian University in Kraków

The thesis concentrates on various results around variational and hemivariational inequalities, or combination of them, sometimes coupled with evolution equations. It consists of five published papers:

(I) Migórski, Stanisław; Zeng, Shengda. Hyperbolic hemivariational inequalities controlled by evolution equations with application to adhesive contact model. Nonlinear Anal. Real World Appl. 43 (2018), 121–143.

(II) Migórski, Stanisław; Zeng, Shengda. Penalty and regularization method for variational-hemivariational inequalities with application to frictional contact. ZAMM Z. Angew. Math. Mech. 98 (2018), no. 8, 1503–1512.

(III) Migórski, Stanisław; Zeng, Shengda. A Class of Generalized Evolutionary Problems Driven by Variational Inequalities and Fractional Operators, Set-Valued Var. Anal (2018). https://doi.org/10.1007/s11228-018-0502-7

(IV) Migórski, Stanisław; Khan, Akhtar A.; Zeng, Shengda. Inverse problems for nonlinear quasi-variational inequalities with an application to implicit obstacle problems of p-Laplacian type. Inverse Problems 35 (2019), no. 3, 035004, 14 pp.

(V) Migórski, Stanisław; Sofonea, Mircea; Zeng, Shengda. Well-posedness of history-dependent sweeping processes. SIAM J. Math. Anal. 51 (2019), no. 2, 1082–1107.

The articles are complemented with the statements of the co-authors, which certify that the contribution of the doctoral student was around 60-70 %, with en exception of the last

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paper, where it was 55%. All the papers were published in good (however not outstanding) journals. They are co-authored by the supervisor of the thesis, which normally would be a weakness. However Shengda Zeng has a long publication track (25 papers), and among them numerous are not co-authored by the supervisor, which proves the ability of Mr Zeng to independent research.

The considered problems are always presented from two perspectives. Firstly, an abstract problem is formulated and various questions of well-posedness are considered and then it is followed by a physical example fitting in the presented theory.

The paper (I) concerns hyperbolic hemivariational inequality. One of the parameters appearing in the inequality (referred as adhesion) is a solution to an ordinary differential equation in a Banach space. The whole framework of hemivariational inequalities extends the standard tools of convex analysis. It is related with situation when instead of having convex potentials we work with locally Lipschitz functions. It has been developed for the last 30 years, and thus these generalisations can be nowadays already called standard. The main concern of this paper is showing global-in-time existence result. The authors underline that using the Rothe method is a novelty for coupled problems of hemivariational inequality and a differential equation, however it is not underlined why particularly it has advantage over other methods. As a byproduct, the authors improve the existence result for a hemivariational inequality without dependence on adhesion, i.e. they manage to extend the result for the case when no smallness assumption for the data is needed.

The second part of the paper refers to applications of the obtained result to a dynamic viscoelastic frictional contact problem with adhesion. I regret that only one example is provided, which is in fact a remark not only concerning this paper, but also most of others. Providing any abstract framework with only one example may always be questionable, whether it is worth to built a general theory instead of considering just one particular system, what may always allow to benefit from particular structure. To avoid such doubts, it is also worth to convince the reader that the applicability is wider.

The second part of the dissertation - paper (II) provides existence and uniqueness for some class of variational-hemivariational inequalities of elliptic type. The same inequality was considered in the earlier paper by Migórski, Ochal and Sofonea (reference [19] in (II)) and existence of solutions was shown. Contrary to this paper, in the thesis the authors study different approximation to the original problem. Instead of applying a penalty method

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only, additional regularisation was introduced. The main result is showing the existence of solutions to approximated problem and the limit passage to the original one. First it is not clear why the authors are interested in showing existence of solutions of the same problem, only with using additional approximation. This however becomes clear in the applicational part. Indeed, it appears that the weak solution of the nonlinear elastic contact problem with the unilateral constraint and the Coulomb friction law can be approximated by the solution of a nonlinear elastic contact problem without unilateral constraint and with the regularised Coulomb friction law. Another importance of the result is the numerical approximation of the problem.

The third paper concerns variational inequalities involving fractional time derivatives. The authors claim that fractional calculus has been of great interest recently, which indeed may be the truth. However the main reason for this popularity is that it opens a way to transfer (without a significant effort) classical results for ODEs to equations involving fractional derivatives. However there are no physical laws, which would lead equations involving e.g. Caputo derivative. For this reason I do not appreciate such kind of results and I do not understand why the PhD candidate having such a choice of papers chose this paper to be a part of the dissertation. It is indeed noteworthy that this is the only paper that does not contain the applicational part.

Paper (IV) concentrates on finding a solution to an inverse problem. This result extends an earlier paper by Khan and Motreanu (refernece [18] in the paper). Both the papers concern quasi-variational inequalities, i.e., such that the constraint sets directly depend on the unknown state variable. The novelty of (IV) is that the assumptions on the map associated to the quasi-variational inequality differ. Moreover, the investigations are transferred from Hilbert to Banach spaces. Finally a particular example of an operator is considered (p-Laplacian) and the developed approach is applied.

The last result - paper (V) - was published in a very good journal SIAM J. Math. Anal. This article is quite puzzling for me. I am not familiar with the notion of sweeping processes. In the paper the authors formulate problem (1.1), which explains what they mean by the sweeping process. However then it is followed by rather vague description that such problems are used to describe various mathematical models arising in mechanics and engineering. I feel a lack of presentation a motivation for these studies. The authors present here existence, uniqueness and stability results for a new class of sweeping processes. As in most of the cases, the framework is presented to be applicable in contact mechanics. However

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it is not at all clear to me why the novelties of this paper, e.g. having an operator Lipschitz continuous instead of linear, nonnegative and symmetric is important in the applicational part. There are no examples of particular materials (and references) showing that some physical properties correspond to the importance of these improvements.

Summarising, I have a positive opinion of the doctoral thesis of Shengda Zeng. He proved to be really fluent with the tools of nonlinear functional analysis and applications in partial differential equations. The thesis consists of five papers, which are well written and concern rather classical field. The candidate appears to be able to successfully collaborate with different mathematicians. The submitted thesis meets all the customary and formal requirements posed to doctoral dissertations and I request for admitting Shengda Zeng to the next stages of the doctoral procedure.

Yours sincerely,

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