

There are two main topics of this thesis. The first topic are the Diophantine equations with separated variables. Second topic are automatic and regular sequences.

In the first chapter we generalize theorem of Peter, Pinter and Schinzel and theorem of Kreso. Cited theorems gives finiteness of the set of integer solutions to the equation  $f(x) = g(y)$ , where  $f, g$  are trinomials. We generalize these theorems to quadrimomials.

The second chapter is about the Stern's diatomic sequence. We give a solution to the problem, which asks to compute certain limit which describes asymptotic behaviour of the Stern's sequence.

The third is about the Stern's polynomials. The set of roots of Stern's polynomials is computed. We describe the set of consecutive polynomials with equal degrees, and show some theorems concerning symmetric polynomials.

In the fourth chapter we deal with sequences related to the Prouhet-Thue-Morse sequence (PTM). We prove that the characteristic sequence of ones in the inverse of PTM sequence is 2-regular, whilst the characteristic sequence of zeros is not  $k$ -regular for any  $k$ . We concern the sequence defined as a convolution of two copies of the PTM sequence. We use a generalization of the Calkin-Wilf tree to compute set of values of this sequence.

Key phrases: Bilu-Tichy Theorem, Diophantine equations, decompositions of polynomials, automatic sequences, regular sequences, Stern's sequence, Prouhet-Thue-Morse sequence, Stern's polynomials.